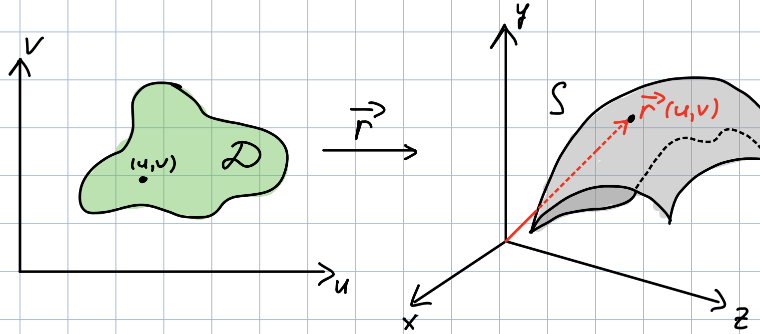
True or false?

$f(x,y,z)$ - function

1) $\iint_S f(x,y,z)\,dS = \iint_D f(\vec{r}(u,v))(r_u \times r_v)\,dA$



Q: How surface integrals of a vector field and a function are related to each other?

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2) Let C be a curve given by $\vec{r}(t)$

(a) $\int_C f(x,y,z)\,ds = f(\vec{r}(b)) - f(\vec{r}(a))$

(b) $\int_C f(x,y,z)\,ds = \int_C f(x,y,z)\,dx + f(x,y,z)\,dy + f(x,y,z)\,dz$

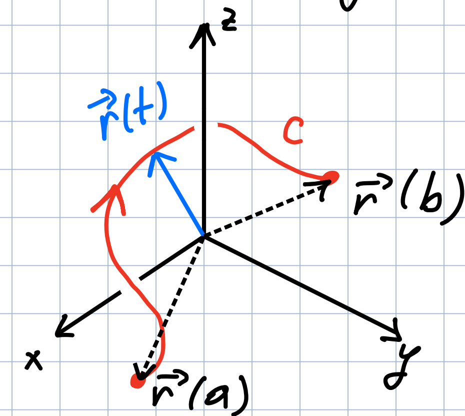
(c) Let $f(x,y,z) = 2x^2 + 3y - z^4$, then

$$\int_C f \cdot d\vec{r} = \int_C 2x^2\,dx + 3y\,dy - z^4\,dz$$

(d) $\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$

$\vec{F} = \langle P, Q, R \rangle$ - vector field

$C: \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$



$f(x,y,z)$ - function

Q: How one can evaluate $\int_C 2x^2\,dx + 3y\,dy - z^4\,dz$ if $\vec{r}(t) = \langle t^2, 2t+2, t\rangle$?
$0 \le t \le 2$

3) $\oint_C \vec{F}\cdot d\vec{r} = \iint_S \vec{F}\cdot d\vec{S}$

Q: How given an orientation of the surface one should orient the boundary curve in Stoke's thm?

Problem: Convert to cylindrical coordinates

$$I = \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy$$ and compute.

Exercises:

1) E is the solid bounded by $y = x^2$, $z = 0$, $y + 2z = 4$. Express the integral $\iiint_E f(x,y,z)\,dV$ as an iterated integral.

2) Sketch the solid whose volume is given by the iterated integral

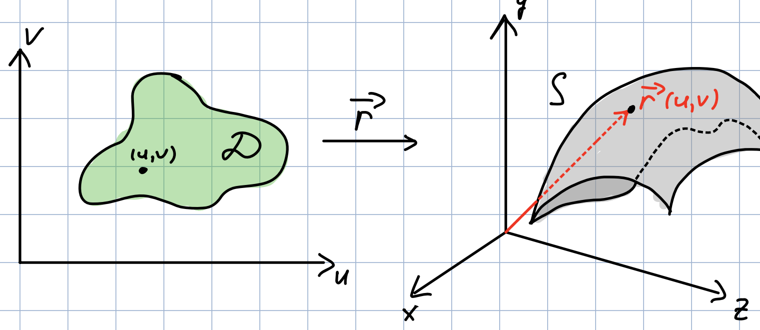
a) $\int_0^1\int_0^{1-x}\int_0^{2-2z} dy\,dz\,dx$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^2\int_0^{r^2} r\,dz\,dr\,d\theta$

Solutions

True or false?

1) $\iint\limits_S f(x,y,z)\,dS = \iint\limits_D F(\vec{r}(u,v))(r_u \times r_v)\,dA$



FALSE

$$\iint\limits_S f(x,y,z)\,dS = \iint\limits_D f(\vec{r}(u,v))\underbrace{|\vec{r}_u \times \vec{r}_v|\overbrace{dA}^{du\,dv}}_{dS}$$

a surface integral of a FUNKTION

$$\iint\limits_S \vec{F}\cdot d\vec{S} = \iint\limits_D \vec{F}(\vec{r}(u,v))\cdot\underbrace{(\vec{r}_u \times \vec{r}_v)\overbrace{dA}^{du\,dv}}_{\vec{n}\,dS}$$

a surface integral of a VECTOR FIELD

Q: How surface integrals of a vector field and a function are related to each other?

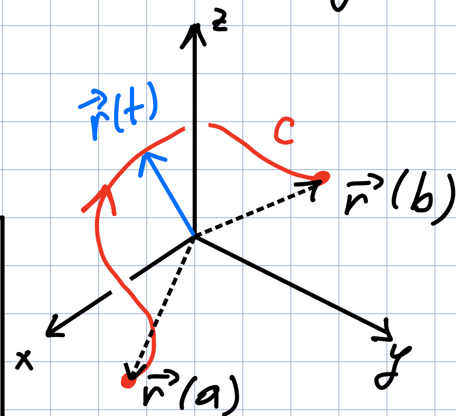
$$\iint\limits_S \vec{F}\cdot d\vec{S} \overset{def}{=} \iint\limits_S \overbrace{\vec{F}\cdot\vec{n}}^{f(x,y,z)}\,dS$$

↑ unit normal vector field for S

2) Let C be a curve given by $\vec{r}(t)$

$\vec{F} = \langle P, Q, R \rangle$ - vector field

$C: \ \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$



$f(x,y,z)$ - function

(a) $\displaystyle\int_C f(x,y,z)\,ds = f(\vec{r}(b)) - f(\vec{r}(a))$

FALSE $\qquad \displaystyle\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

(b) $\displaystyle\int_C f(x,y,z)\,ds = \int_C f(x,y,z)\,dx + f(x,y,z)\,dy + f(x,y,z)\,dz$

FALSE

$$\int_C f(x,y,z)\,ds = \int_a^b f(x(t),y(t),z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$$

(c) Let $f(x,y,z) = 2x^2 + 3y - z^4$, then

$$\int_C f \cdot d\vec{r} = \int_C 2x^2\,dx + 3y\,dy - z^4\,dz$$

FALSE $\qquad f \cdot d\vec{r}$ - doesn't make scense

$\displaystyle\int_C \vec{F} \cdot d\vec{r} = \int_C 2x^2\,dx + 3y\,dy - z^4\,dz$

for $\vec{F} = \langle 2x^2, 3y, -z^4 \rangle$

(d) $\displaystyle\int_C \vec{F} \cdot d\vec{r} = \int_a^b \underbrace{\vec{F}(\vec{r}(t))}_{\vec{F}(x(t),y(t),z(t))} \cdot \vec{r}'(t)\,dt$

TRUE

Q: How one can evaluate $\int_C 2x^2\,dx + 3y\,dy - z^4\,dz$ if $\vec{r}(t) = \langle t^2, 2t+2, t\rangle$?

$0 \le t \le 2$

$$\int_C F(x,y,z)\,dx = \int_a^b F(x(t), y(t), z(t))\, x'(t)\,dt$$

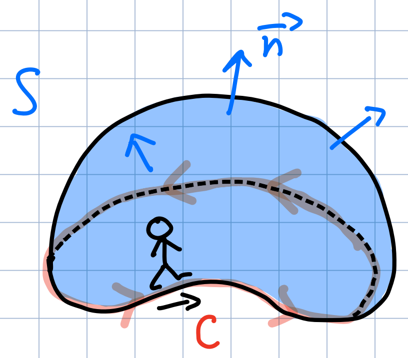
$$\int_C 2x^2\,dx + 3y\,dy - z^4\,dz = \int_0^2 2(t^2)^2 2t + 3(2t+2)(2) - t^4(1)\,dt = \int_0^2 (4t^5 - t^4 + 12t + 12)\,dt = \ldots$$

3) $\oint_C \vec{F}\cdot d\vec{r} = \iint_S \vec{F}\cdot \vec{dS}$

FALSE

Stokes thm: $\oint_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{curl}\vec{F}\cdot\vec{dS}$

Q: How given an orientation of the surface one should orient the boundary curve in Stokes thm?



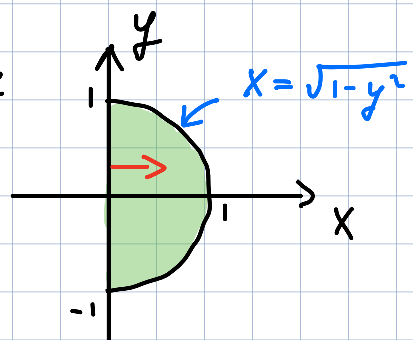
Ex: Convert to cylindrical

$$I=\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy \quad \text{and compute.}$$

Sol:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy=\iiint_E xyz\,dV$$

Solid E above



with $x^2+y^2\le z\le\sqrt{x^2+y^2}$

in cylindrical coordinates:

$$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$$

$$0\le r\le 1$$

$$r^2\le z\le r$$

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{1}\underbrace{\int_{r^2}^{r}\underbrace{r\cos\theta\cdot r\sin\theta\cdot z}_{\frac{1}{2}zr^2\sin 2\theta}\,r\,dz}_{\frac{1}{4}z^2r^3\sin 2\theta\Big|_{z=r^2}^{z=r}}\,dr\,d\theta=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\underbrace{\int_{0}^{1}\frac{r^2-r^4}{4}r^3\sin 2\theta\,dr}_{\frac{1}{4}\sin 2\theta\left(\frac{1}{6}r^6-\frac{1}{8}r^8\right)\Big|_{r=0}^{r=1}}\,d\theta$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{4}\underbrace{\left(\frac{1}{6}-\frac{1}{8}\right)}_{\frac{1}{24}}\sin 2\theta\,d\theta=\frac{1}{96}\,\frac{-1}{2}\cos 2\theta\Big|_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}}$$

E is the solid bounded by $y=x^2$, $z=0$, $y+2z=4$. Express the integral $\iiint_E f(x,y,z)\,dV$ as an iterated integral.

$$\iiint_E f(x,y,z)\,dV = \int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{2-\frac{y}{2}} f\,dz\,dy\,dx$$

$$= \int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{2-\frac{y}{2}} f(x,y,z)\,dz\,dx\,dy$$

$$= \int_{0}^{2}\int_{0}^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f\,dx\,dy\,dz$$

$$= \int_{0}^{4}\int_{0}^{2-\frac{y}{2}}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dz\,dy$$

$$= \int_{-2}^{2}\int_{0}^{2-\frac{x^2}{2}}\int_{x^2}^{4-2z} f\,dy\,dz\,dx$$

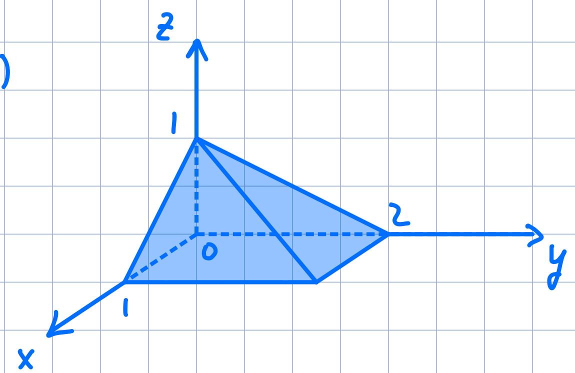
$$= \int_{0}^{2}\int_{-\sqrt{4-2x}}^{\sqrt{4-2x}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dx\,dz$$

Sketch the solid whose volume is given by the iterated integral

a) $\int_0^1\int_0^{1-x}\int_0^{2-2z} dy\,dz\,dx$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^2\int_0^{r^2} r\,dz\,dr\,d\theta$

(a)



(b)